

20/3/20



K23U 0513

Reg. No. :

Name :

**VI Semester B.Sc. Degree (CBCSS-OBE-Regular/Supplementary/
Improvement) Examination, April 2023
(2019 and 2020 Admissions)
CORE COURSE IN MATHEMATICS
6B10 MAT : Real Analysis – II**

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

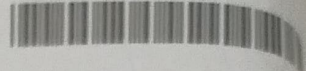
1. State second form of the fundamental theorem of integral calculus.
2. State Lebesgue's integrability criterion.
3. Evaluate $\int_0^{\infty} \frac{dx}{1+x^2}$.
4. Evaluate $\int_0^{\infty} x^4 e^{-x} dx$.
5. Find the limit of the sequence of function $f_n(x) = x^n$ on $[0, 1]$.

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Prove that $f(x) = \sqrt{x}$ is uniformly continuous on $[1, \infty)$.
7. State nonuniform continuity criteria.
8. If $f : A \rightarrow \mathbb{R}$ is a Lipschitz function, then prove that f is uniformly continuous on A .
9. Prove that every constant function on $[a, b]$ is in $\mathcal{R}[a, b]$.
10. If $f(x) = x^2$, for $x \in [0, 4]$, calculate the Riemann sum with respect to the partition $\mathcal{P} = \{0, 1, 2, 4\}$ with tags at the left end points of the sub intervals.
11. Prove that the function $d(x, y) = |x - y|$ is a metric on \mathbb{R} .
12. Define closed set in a metric space. Give an example.
13. Investigate the convergence of $\int_0^1 \frac{1}{1-x} dx$.
14. Prove that $\int_1^{\infty} \frac{(1-e^{-x})}{x} dx$ diverges.

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15. Evaluate $\int_1^{\infty} \sqrt{x} e^{-x^2} dx$.

16. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

PART - C

Answer any four questions. Each question carries four marks.

17. Show that if f and g are uniformly continuous on $A \subseteq \mathbb{R}$ and if they are both bounded on A , then their product $f g$ is uniformly continuous on A .

18. If $f \in \mathcal{R}[a, b]$, then prove that f is bounded.

19. Evaluate $\int_0^1 \frac{dx}{(x-1)^{\frac{2}{3}}}$.

20. Prove that $B(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$.

21. Prove that $\Gamma m \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \cdot \Gamma(2m)$.

22. Show that the sequence of functions $\left(\frac{x^n}{1+x^n}\right)$ does not converge uniformly on $[0, 2]$.

23. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \rightarrow \mathbb{R}$. Then prove that f is continuous on A .

PART - D

Answer any two questions. Each question carries 6 marks.

24. State and prove continuous extension theorem.

25. Prove that a function $f \in \mathcal{R}[a, b]$ if and only if for every $\epsilon > 0$ there exists $\eta_\epsilon > 0$ such that if \dot{P} and \dot{Q} are any two tagged partitions of $[a, b]$ with $\|\dot{P}\| < \eta_\epsilon$ and $\|\dot{Q}\| < \eta_\epsilon$, then $|S(f, \dot{P}) - S(f, \dot{Q})| < \epsilon$.

26. Prove that if $f : [a, b] \rightarrow \mathbb{R}$ is monotone on $[a, b]$, then $f \in \mathcal{R}[a, b]$.

27. State and prove Cauchy criterion for uniform convergence of sequence of functions.